

Towards Minimal S_4 Lepton Flavor Model

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Abstract

We study lepton flavor models with the S_4 flavor symmetry. We construct simple models with smaller numbers of flavon fields and free parameters, such that we have predictions among lepton masses and mixing angles. The model with a S_4 triplet flavon is not realistic, but we can construct realistic models with two triplet flavons, or one triplet and one doublet flavons.

1 Introduction

In particle physics, it is one of most important issues to understand the origin of the hierarchy among quark/lepton masses and their mixing angles. Indeed, there are many free parameters in the standard model including its extension with neutrino mass terms, and most of them are originated from the flavor sector, i.e. Yukawa couplings of quarks and leptons. Recent experiments of the neutrino oscillation can determine neutrino mass squared differences and mixing angles increasing their preciseness [1, 2, 3, 4, 5]. This indicates large mixing angles, which are completely different from the quark mixing ones. In particular, the tri-bimaximal mixing is one of interesting Ansatz in the lepton sector [6, 7, 8, 9].

Non-Abelian flavor symmetries, in particular non-Abelian discrete symmetries, could explain such large mixing angles [10]. For example, by use of the A_4 flavor symmetry, the tri-bimaximal mixing of leptons has been derived [11, 12, 13, 14, 15]. Furthermore, phenomenologically interesting aspects of A_4 flavor models have been studied [16]-[79]. Another interesting flavor symmetry is the S_4 symmetry [80, 81, 82, 83]. One can realize the exact tri-bimaximal neutrino mixing in S_4 flavor models [84, 85, 86, 87, 88, 89, 90]. The S_4 flavor symmetry can lead other interesting aspects such as realistic quark mass matrices, a grand unified theory, etc [91]-[104].

The tri-bimaximal mixing is quite interesting Ansatz at a certain level. For θ_{13} , we have its upper bound. It is a current experimental target to measure a finite value of θ_{13} , and a finite value of θ_{13} would be measured in near future.¹ (See also for a global fit analysis of neutrino oscillation data [5], which suggests non-vanishing value for the mixing angle θ_{13} .) It would be straightforward to obtain non-zero θ_{13} by adding correction terms in the models leading to the tri-bimaximal mixing. In this case, we may have no clear prediction on θ_{13} in some models, although we could keep our predictability on other models. At any rate, models would become complicated.

Indeed, most of models include several flavon fields, whose vacuum expectation values (VEVs) break flavor symmetries. In addition, there are many free parameters to derive lepton masses and mixing angles. Thus it is important to study whether models with the minimal or smaller number of flavon fields can lead to realistic results and whether there are models with higher predictability, that is, that the number of free parameters is smaller than the number of observables such as masses and mixing angles. Our purpose here is to study simple models with a small number of flavon fields and a small number of free parameters, such that our models have predictions on masses and mixing angles, e.g. their relations.

In this paper, we consider S_4 as the flavor symmetry and study simple supersymmetric model constructions with the smaller numbers of flavons and free parameters. When the three families correspond to a S_4 triplet, we have smaller number of free parameters. On the other hand, when the three families correspond to a singlet and a doublet, couplings including the S_4 singlet lepton and S_4 doublet are independent of each other. Then, we would have more free parameters. Thus, here we concentrate on the models, in which the three families of both the left and right-handed leptons correspond to S_4 triplets. Obviously the simplest model is the model with only one triplet flavon. However, we show that such models do not lead to realistic results. Hence, we add a S_4 doublet or triplet as the next

¹After this paper was completed, Ref. [105] appeared.

simple models. These models have seven free parameters in the lepton mass matrices. Thus, they have predictions among masses and mixing angles. Furthermore, since the neutrino mass spectrum is determined, the sum of neutrino masses and effective mass of double beta decay are also predicted. These predictions would be useful to search a hint of non-Abelian flavor symmetry S_4 .

This paper is organized as follows. In section 2, we study the simple model with one S_4 triplet flavon. Such a model is not realistic. In section 3 we study the model with one triplet and one doublet flavon fields, that is, model III. In section 4 we study the model with two triplet flavon fields, that is, model IV. The models III and IV are realistic and have predictions among lepton masses and mixing angles. In section 5, we give a comment on the model with the $\Delta(54)$ flavor symmetry, which is quite similar to the model IV. Section 6 is devoted to the summary.

2 Model with a triplet flavon

The simplest model is the model with a triplet flavon. In this section, we study such two models and show we can not obtain realistic results.

2.1 Model I

	$(\ell_e, \ell_\mu, \ell_\tau)$	(e^c, μ^c, τ^c)	$H_{u,d}$	(χ_1, χ_2, χ_3)
S_4	3	3	1	3

Table 1: Matter content and charge assignment of model I.

We first consider the simplest model among all other S_4 models, i.e. model I. Each of left-handed lepton doublets and right-handed charged leptons are assigned to S_4 triplet 3 and additional flavon fields (χ_1, χ_2, χ_3) are also assigned to the same triplet. The up and down sectors of electroweak Higgs fields are S_4 trivial singlets. These S_4 representations are shown in Table 1. In this model, the superpotential of charged leptons is written by

$$w_e = y_1^e (e^c \ell_e + \mu^c \ell_\mu + \tau^c \ell_\tau) H_d + y_2^e ((\mu^c \ell_\tau + \tau^c \ell_\mu) \chi_1 + (e^c \ell_\tau + \tau^c \ell_e) \chi_2 + (e^c \ell_\mu + \mu^c \ell_e) \chi_3) H_d / \Lambda. \quad (1)$$

For the neutrino sector, we have

$$w_\nu = y_1^\nu (\ell_e \ell_e + \ell_\mu \ell_\mu + \ell_\tau \ell_\tau) H_u H_u / \Lambda + y_2^\nu ((\ell_\mu \ell_\tau + \ell_\tau \ell_\mu) \chi_1 + (\ell_e \ell_\tau + \ell_\tau \ell_e) \chi_2 + (\ell_e \ell_\mu + \ell_\mu \ell_e) \chi_3) H_u H_u / \Lambda^2. \quad (2)$$

The VEVs of scalar fields are given by

$$\langle H_{u,d} \rangle = v_{u,d}, \quad \langle \chi_n \rangle = \alpha_n \Lambda. \quad (3)$$

We reparametrize the VEVs of χ_i for $i = 1, 2, 3$ as

$$\langle (\chi_1, \chi_2, \chi_3) \rangle = \alpha_1 \Lambda (1, r, r'). \quad (4)$$

Then mass matrices become

$$\begin{aligned}
M_e &= y_1^e v_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^e \alpha_1 v_d \begin{pmatrix} 0 & r' & r \\ r' & 0 & 1 \\ r & 1 & 0 \end{pmatrix}, \\
M_\nu &= y_1^\nu \frac{v_u^2}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^\nu \alpha_1 \frac{v_u^2}{\Lambda} \begin{pmatrix} 0 & r' & r \\ r' & 0 & 1 \\ r & 1 & 0 \end{pmatrix}.
\end{aligned} \tag{5}$$

The off-diagonal elements of the neutrino mass matrix are the same as the ones of charged leptons. Indeed, we can rewrite the mass matrix of neutrinos as

$$M_\nu = \left(y_1^\nu - \frac{y_1^e y_2^\nu}{y_2^e} \right) \frac{v_u^2}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{y_2^\nu v_u^2}{y_2^e v_d \Lambda} M_e. \tag{6}$$

Then, one can not realize large mixing angles. With a non-vanishing CP-phase, the mixing matrix does not need to be trivial, but we cannot obtain large mixing angles indicated by experiments of neutrino oscillation. We could introduce a Z_N symmetry such that it allows either y_2^e or y_2^ν . In this case, one could not realize realistic mass eigenvalues.

2.2 Model II

	$(\ell_e, \ell_\mu, \ell_\tau)$	(e^c, μ^c, τ^c)	$H_{u,d}$	χ_1	(χ_2, χ_3, χ_4)
S_4	3	3'	1	1'	3

Table 2: Matter content and charge assignment of model II.

Here, we discuss another model with a S_4 triplet flavon as model II. As indicated by the model I, if the off-diagonal elements of charged leptons and neutrinos are the same, realistic lepton mixing cannot be obtained. Another candidate for the simplest model is given by changing the S_4 charge assignment. Lepton doublets are assigned to 3 while right handed-charged leptons are assigned to 3' of S_4 . In addition, we consider S_4 singlet flavon χ_1 with the charge 1' and triplet flavon (χ_2, χ_3, χ_4) with 3'. These S_4 representations are shown in Table 2. Then the flavor symmetric superpotential becomes

$$\begin{aligned}
w_e &= y_1^e (e^c \ell_e + \mu^c \ell_\mu + \tau^c \ell_\tau) \chi_1 H_d \\
&\quad + y_2^e ((\tau^c \ell_\mu - \mu^c \ell_\tau) \chi_2 + (e^c \ell_\tau - \tau^c \ell_e) \chi_3 + (\mu^c \ell_e - e^c \ell_\mu) \chi_4) H_d / \Lambda,
\end{aligned} \tag{7}$$

for charged leptons and

$$\begin{aligned}
w_\nu &= y_1^\nu (\ell_e \ell_e + \ell_\mu \ell_\mu + \ell_\tau \ell_\tau) H_u H_u / \Lambda \\
&\quad + y_2^\nu ((\ell_\mu \ell_\tau + \ell_\tau \ell_\mu) \chi_2 + (\ell_e \ell_\tau + \ell_\tau \ell_e) \chi_3 + (\ell_e \ell_\mu + \ell_\mu \ell_e) \chi_4) H_u H_u / \Lambda^2,
\end{aligned} \tag{8}$$

for neutrinos. Vacuum expectation values are given by

$$\langle H_{u,d} \rangle = v_{u,d}, \quad \langle \chi_1 \rangle = \alpha_1 \Lambda, \quad \langle (\chi_2, \chi_3, \chi_4) \rangle = \alpha_2 \Lambda (1, r, r'). \tag{9}$$

Then, mass matrices are obtained

$$\begin{aligned}
M_e &= y_1^e \alpha_1 v_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^e \alpha_2 v_d \begin{pmatrix} 0 & -r' & r \\ r' & 0 & -1 \\ -r & 1 & 0 \end{pmatrix}, \\
M_\nu &= y_1^\nu \frac{v_u^2}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^\nu \alpha_2 \frac{v_u^2}{\Lambda} \begin{pmatrix} 0 & r' & r \\ r' & 0 & 1 \\ r & 1 & 0 \end{pmatrix}.
\end{aligned} \tag{10}$$

Let us consider the limit $m_e = 0$. Then, the determinant of the charged lepton mass matrix can be vanishing when $y_1^e \alpha_1 / y_2^e \alpha_2 = 0, \pm \sqrt{-1 - r^2 - r'^2}$. For the first case, we have $m_\mu = m_\tau$ and for the other cases, $2m_\mu = m_\tau$. Then this model cannot lead the realistic mass spectrum of charged leptons.

3 Model III

In the previous section, it was shown that the models with a S_4 triplet flavon does not lead to realistic results. Thus, the next step is to add another flavon with non-trivial S_4 representations. In this section we add a S_4 doublet flavon, and in the next section we add a S_4 triplet flavon.

3.1 Mass matrices

	$(\ell_e, \ell_\mu, \ell_\tau)$	(e^c, μ^c, τ^c)	$H_{u,d}$	χ_0	χ_1	(χ_2, χ_3)	(χ_4, χ_5, χ_6)
S_4	3	3'	1	1	1	2	3'
Z_3	1	1	0	0	1	1	1

Table 3: Matter content and charge assignment of model III.

In this model, we add one S_4 doublet. Then, we can fit experimental values. The charge assignments for leptons and flavons are summarized in Table 3. The field χ_0 is added to realize a proper pattern of the vacuum alignment as will be discussed. To make stronger prediction, we assume there is no mixing from the neutrino sector which is realized in the charge assignment with the Z_3 symmetry. The superpotential of charged leptons is

$$\begin{aligned}
w_e &= y_1^e (e^c \ell_e + \mu^c \ell_\mu + \tau^c \ell_\tau) \chi_1 H_d / \Lambda \\
&+ y_2^e \left(\frac{1}{\sqrt{2}} (\mu^c \ell_\mu - \tau^c \ell_\tau) \chi_3 + \frac{1}{\sqrt{6}} (2e^c \ell_e - \mu^c \ell_\mu - \tau^c \ell_\tau) \chi_2 \right) H_d / \Lambda \\
&+ y_3^e ((\tau^c \ell_\mu + \mu^c \ell_\tau) \chi_3 + (e^c \ell_\tau + \tau^c \ell_e) \chi_4 + (\mu^c \ell_e + e^c \ell_\mu) \chi_5) H_d / \Lambda.
\end{aligned} \tag{11}$$

Similarly, the superpotential of neutrinos is obtained

$$\begin{aligned}
w_\nu &= y_1^\nu (\ell_e \ell_e + \ell_\mu \ell_\mu + \ell_\tau \ell_\tau) \chi_1 H_u H_u / \Lambda^2 \\
&+ y_2^\nu \left(\frac{1}{\sqrt{2}} (\ell_\mu \ell_\mu - \ell_\tau \ell_\tau) \chi_2 + \frac{1}{\sqrt{6}} (-2\ell_e \ell_e + \ell_\mu \ell_\mu + \ell_\tau \ell_\tau) \chi_3 \right) H_u H_u / \Lambda.
\end{aligned} \tag{12}$$

VEVs are denoted by

$$\langle H_{u,d} \rangle = v_{u,d}, \quad \langle \chi_n \rangle = \alpha_n \Lambda. \quad (13)$$

The vacuum alignment is assumed to be

$$\langle (\chi_2, \chi_3) \rangle = \alpha_2 \Lambda(1, r), \quad \langle (\chi_4, \chi_5, \chi_6) \rangle = \alpha_4 \Lambda(1, r', r'). \quad (14)$$

Then, mass matrices are written

$$M_e = y_1^e \alpha_1 v_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^e \alpha_2 v_d \begin{pmatrix} \frac{2}{\sqrt{6}} & 0 & 0 \\ 0 & \frac{r}{\sqrt{2}} - \frac{1}{\sqrt{6}} & 0 \\ 0 & 0 & -\frac{r}{\sqrt{2}} - \frac{1}{\sqrt{6}} \end{pmatrix} + y_3^e \alpha_4 v_d \begin{pmatrix} 0 & r' & r' \\ r' & 0 & 1 \\ r' & 1 & 0 \end{pmatrix}, \quad (15)$$

$$M_\nu = y_1^\nu \alpha_1 \frac{v_u^2}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^\nu \alpha_2 \frac{v_u^2}{\Lambda} \begin{pmatrix} -\frac{2r}{\sqrt{6}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} \end{pmatrix}.$$

Suppose that the charged lepton mass matrix is diagonalized by U_e . The neutrino mass matrix in the basis of the diagonal charged lepton mass matrix is written

$$U_e^T M_\nu U_e = \frac{y_3^\nu}{y_3^e} \frac{v_u^2}{v_d \Lambda} U_e^T U_e \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} + (y_1^\nu - \frac{y_3^\nu y_1^e}{y_3^e}) \frac{v_u^2}{\Lambda} U_e^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} U_e$$

$$+ (y_2^\nu - \frac{y_2^e y_3^\nu}{y_3^e}) \alpha_1 \frac{v_u^2}{\Lambda} U_e^T \begin{pmatrix} -\frac{2r}{\sqrt{6}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} \end{pmatrix} U_e. \quad (16)$$

In this case, by introducing the S_4 doublet flavon, we have more free parameters in M_e compared with the mass matrices in the previous section. Then we can obtain realistic values of charged lepton masses and two large mixing angles.

There are five parameters for the charged lepton mass matrix so that they can be fixed by giving two mixing angles of leptons and charged lepton masses. The other mixing angle is determined. After fixing the parameters, the neutrino mass matrix has two degrees of freedom which can be determined by mass squared differences of neutrinos. For instance, when we give $\sin^2 \theta_{12}^{\text{MNS}} = 1/3$, $\sin^2 \theta_{23}^{\text{MNS}} = 1/2$, we obtain θ_{13}^{MNS} and the mass spectrum of neutrinos. Such angles as well as charged lepton mass ratios are realized when $r' = 0.567$, $y_1^e/y_3^e \alpha_3 = 0.857$, $y_2^e/y_3^e \alpha_3 = -0.225$, and $r = -2.83$, then we obtain

$$M_e \approx y_3 \alpha_4 v_d \begin{pmatrix} 0.338 & 0.567 & 0.567 \\ 0.567 & 0.957 & 1 \\ 0.567 & 1 & 1.27 \end{pmatrix}, \quad U_e^T \approx \begin{pmatrix} 0.813 & -0.575 & 0.0895 \\ -0.460 & -0.541 & 0.704 \\ -0.357 & -0.614 & -0.704 \end{pmatrix}. \quad (17)$$

The predicted value for $\sin \theta_{13}^{\text{MNS}}$ is $\sin \theta_{13}^{\text{MNS}} \approx 0.0895$. For neutrino masses, we have

$$m_{\nu_1} = y_2^\nu \alpha_1 \frac{v_u^2}{\Lambda} \left(\frac{y_1^\nu \alpha_1}{y_2^\nu \alpha_2} - \frac{2r}{\sqrt{6}} \right), \quad m_{\nu_2} = y_2^\nu \alpha_1 \frac{v_u^2}{\Lambda} \left(\frac{y_1^\nu \alpha_1}{y_2^\nu \alpha_2} + \frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} \right),$$

$$m_{\nu_3} = y_2^\nu \alpha_1 \frac{v_u^2}{\Lambda} \left(\frac{y_1^\nu \alpha_1}{y_2^\nu \alpha_2} - \frac{1}{\sqrt{2}} + \frac{r}{\sqrt{6}} \right). \quad (18)$$

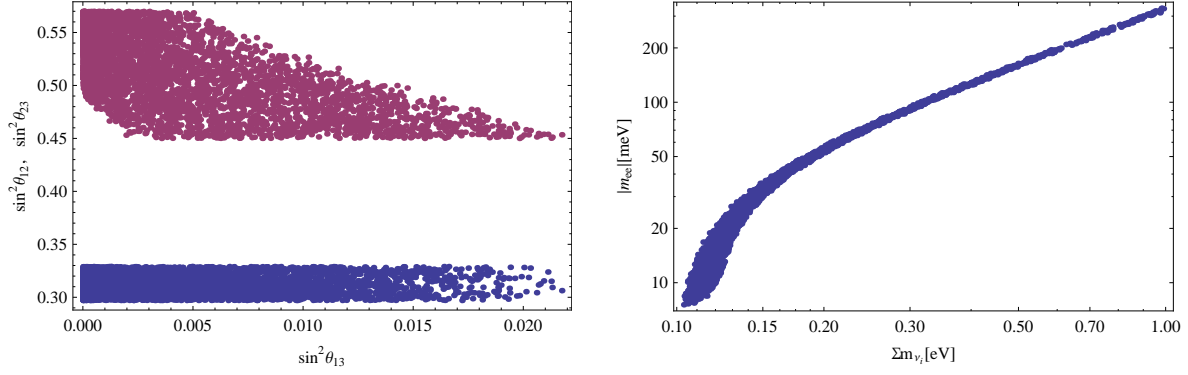


Figure 1: Predicted values of mixing angles (left) and neutrino masses (right). For the left figure, blue dots (lower dots) denote $\sin^2 \theta_{13}$ – $\sin^2 \theta_{12}$ plane and red dots (upper dots) denote $\sin^2 \theta_{13}$ – $\sin^2 \theta_{23}$ plane.

Mass squared differences of atmospheric and solar are obtained by assuming normal hierarchy. Writing $\text{Arg}(\frac{y_1^\nu \alpha_1}{y_2^\nu \alpha_2}) = a$, we have

$$\frac{y_1^\nu \alpha_1}{y_2^\nu \alpha_2} \approx \frac{1.93e^{2ia}}{1 + e^{2ia}}, \quad |y_2^\nu| \alpha_1 \frac{v_u^2}{\Lambda} \approx 0.0201[\text{eV}]. \quad (19)$$

From them, the lowest value of the sum of neutrino mass becomes $\sum m_{\nu_i} \approx 0.113\text{eV}$ and of the effective mass of double beta decay is $|m_{ee}| \approx 8.05\text{meV}$.

In numerical calculation, we input θ_{12}^{MNS} and θ_{23}^{MNS} within 1σ range of [5] for the case of normal mass hierarchy:

$$\begin{aligned} \Delta m_{31}^2 &= (2.36 - 2.54) \times 10^{-3} \text{eV}^2, & \Delta m_{21}^2 &= (7.41 - 7.79) \times 10^{-5} \text{eV}^2, \\ \sin^2 \theta_{12} &= 0.297 - 0.329, & \sin^2 \theta_{23} &= 0.45 - 0.57. \end{aligned} \quad (20)$$

With these values for θ_{12}^{MNS} and θ_{23}^{MNS} , we obtain θ_{13}^{MNS} , $\sum m_{\nu_i}$, and $|m_{ee}|$ for each value, as indicated by Fig. 1. Important predictions of this model are the correlation of $\sin^2 \theta_{13}$ – $\sin^2 \theta_{23}$ with narrow band and allowed region of $\sum m_{\nu_i}$ – $|m_{ee}|$ plane. Considering improvement of experiments in future, prediction would be stronger, depending on the parameter region of input values. Precise measurement of mixing angles can test the model in near future. For neutrino masses, they can be also improved by the precise values of input mixing angles.

3.2 Potential analysis

The superpotential including only the flavon fields is obtained as

$$\begin{aligned} w_s &= \kappa \chi_0^2 + \lambda_1 \chi_1^3 + \lambda_2 \chi_1 (\chi_2^2 + \chi_3^2) + \lambda_3 \chi_1 (\chi_4^2 + \chi_5^2 + \chi_6^2) \\ &\quad + \lambda_4 (3\chi_2^2 \chi_3 - \chi_3^3) + \lambda_5 \left(\frac{1}{\sqrt{2}} \chi_2 (\chi_5^2 - \chi_6^2) + \frac{1}{\sqrt{6}} \chi_3 (-2\chi_4^2 + \chi_5^2 + \chi_6^2) \right) \\ &\quad + \eta_1 \chi_0^4 + 6\eta_2 \chi_0 \chi_4 \chi_5 \chi_6. \end{aligned} \quad (21)$$

The conditions of potential minimum read

$$\begin{aligned}
2\kappa\chi_0 + 4\eta_1\chi_0^3 + 6\eta_2\chi_4\chi_5\chi_6 &= 0, \\
3\lambda_1\chi_1^2 + \lambda_2(\chi_2^2 + \chi_3^2) + \lambda_3(\chi_4^2 + \chi_5^2 + \chi_6^2) &= 0, \\
2\kappa_2\chi_2 + \lambda_{12}(\chi_3^3 - 3\chi_3\chi_4^2) + 6\lambda_{14}\chi_5\chi_6\chi_7 &= 0, \\
2\lambda_2\chi_1\chi_2 + 6\lambda_4\chi_2\chi_3 + \frac{1}{\sqrt{2}}\lambda_5(\chi_5^2 - \chi_6^2) &= 0, \\
2\lambda_2\chi_1\chi_3 + 3\lambda_4(\chi_2^2 - \chi_3^2) + \frac{1}{\sqrt{6}}\lambda_5(-2\chi_4^2 + \chi_5^2 + \chi_6^2) &= 0, \\
2\lambda_3\chi_1\chi_4 - \frac{4}{\sqrt{6}}\lambda_5\chi_3\chi_4 + 6\eta_2\chi_0\chi_5\chi_6 &= 0, \\
2\lambda_3\chi_1\chi_5 + 2\lambda_5\chi_2\chi_5 + \frac{2}{\sqrt{6}}\lambda_5\chi_3\chi_5 + 6\eta_2\chi_0\chi_4\chi_6 &= 0, \\
2\lambda_3\chi_1\chi_6 - 2\lambda_5\chi_2\chi_6 + \frac{2}{\sqrt{6}}\lambda_5\chi_3\chi_6 + 6\eta_2\chi_0\chi_4\chi_5 &= 0.
\end{aligned} \tag{22}$$

Since there are more than six parameters, it is easy to obtain independent values for all VEVs. Note that the alignment with strict relation $\chi_5 = \chi_6$ leads $\lambda_5 = 0$ from the last three equations. Taking this, it automatically makes $\chi_4 = \chi_5 = \chi_6$ if they have non-vanishing VEVs. Then the vacuum alignment of the model must be interpreted as $\chi_5 \approx \chi_6$. Choosing some parameter region of the above superpotential, this relation holds so that the same result can be obtained.

4 Model IV

4.1 Mass matrices

	$(\ell_e, \ell_\mu, \ell_\tau)$	(e^c, μ^c, τ^c)	$H_{u,d}$	χ_1	χ_2	(χ_3, χ_4, χ_5)	(χ_6, χ_7, χ_8)
S_4	3	3	1	1	1	3	3
Z_3	1	0	0	2	1	2	1

Table 4: Matter content and charge assignment of model IV.

Here, we study the model with two S_4 triplet flavons, that is, model IV. In this model, each of charged lepton sector and neutrino sector couples to (different) S_4 triplet flavon. The pattern of mass matrices is the same between the charged leptons and neutrinos. Then the maximal number of parameters is equal to four in each sector. Considering a proper pattern of the vacuum alignment, the model has some prediction. The S_4 representations and Z_3 charges are shown in Table 4. Now let us study the prediction of this model. The superpotential of charged leptons is written by

$$\begin{aligned}
w_e = & y_1^e (e^c \ell_e + \mu^c \ell_\mu + \tau^c \ell_\tau) \chi_1 H_d / \Lambda \\
& + y_2^e ((\tau^c \ell_\mu + \mu^c \ell_\tau) \chi_3 + (e^c \ell_\tau + \tau^c \ell_e) \chi_4 + (\mu^c \ell_e + e^c \ell_\mu) \chi_5) H_d / \Lambda.
\end{aligned} \tag{23}$$

The superpotential including neutrinos is written

$$w_\nu = y_1^\nu (\ell_e \ell_e + \ell_\mu \ell_\mu + \ell_\tau \ell_\tau) \chi_2 H_u H_u / \Lambda^2 + y_2^\nu ((\ell_\mu \ell_\tau + \ell_\tau \ell_\mu) \chi_6 + (\ell_e \ell_\tau + \ell_\tau \ell_e) \chi_7 + (\ell_e \ell_\mu + \ell_\mu \ell_e) \chi_8) H_u H_u / \Lambda^2. \quad (24)$$

The vacuum alignment is assumed to be

$$\langle (\chi_3, \chi_4, \chi_5) \rangle = \alpha_3 \Lambda (1, 1, r), \quad \langle (\chi_6, \chi_7, \chi_8) \rangle = \alpha_7 \Lambda (1, r', r''). \quad (25)$$

Then the mass matrices are given

$$M_e = y_1^e v_d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^e \alpha_3 v_d \begin{pmatrix} 0 & r & 1 \\ r & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (26)$$

$$M_\nu = y_1^\nu \frac{v_u^2}{\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_2^\nu \alpha_7 \frac{v_u^2}{\Lambda} \begin{pmatrix} 0 & r'' & r' \\ r'' & 0 & 1 \\ r' & 1 & 0 \end{pmatrix}.$$

For the charged leptons, there remains the $e - \mu$ symmetry so that

$$U_{12} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{12}^\dagger M_e U_{12} = y_2^e \alpha_3 v_d \begin{pmatrix} a - r & 0 & 0 \\ 0 & a + r & \sqrt{2} \\ 0 & \sqrt{2} & a \end{pmatrix}, \quad (27)$$

where $a = y_1^e \alpha_1 / y_2^e \alpha_2$. Then the mass matrix M_e can be diagonalized by

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & -\sin \theta_{23} \\ 0 & \sin \theta_{23} & \cos \theta_{23} \end{pmatrix}, \quad \tan \theta_{23} = \frac{-r + \sqrt{8 + r^2}}{2\sqrt{2}}, \quad (28)$$

$$U_{23}^{\dagger} U_{12}^{\dagger} M_e U_{12} U_{23}^e = y_2^e \alpha_2 v_d \begin{pmatrix} a - r & 0 & 0 \\ 0 & \frac{1}{2}(2a + r + \sqrt{8 + r^2}) & 0 \\ 0 & 0 & \frac{1}{2}(2a + r - \sqrt{8 + r^2}) \end{pmatrix}.$$

We use the notation of $U_{12}^{\dagger} U_{23}^{\dagger} M_e U_{23}^e U_{12}^e = \text{diag}(m_1^e, m_2^e, m_3^e)$. The mass matrix M_e has three parameters and they can be fixed by masses of charged leptons.

The parameters of the neutrino mass matrix are independent of the ones of charged leptons so that four parameters remain. Using them, we need to fit two mass scales of neutrino oscillations and three mixing angles. Similar to the previous section, giving two mixing angles of the MNS matrix, the other angle and neutrino mass spectrum can be predicted. To fit the parameters, we write

$$M_\nu = U_\nu \begin{pmatrix} m_1^\nu & 0 & 0 \\ 0 & m_2^\nu & 0 \\ 0 & 0 & m_3^\nu \end{pmatrix} U_\nu^\dagger. \quad (29)$$

As an example, assuming $m_1^e = m_e$, $m_2^e = m_\mu$, $m_3^e = m_\tau$, we have

$$a \approx -1 \pm \frac{9m_\mu}{4m_\tau}, \quad r \approx 1 \mp \frac{9m_\mu}{4m_\tau}. \quad (30)$$

For neutrino masses, we assume $m_1^\nu = m_{\nu_1}$, $m_2^\nu = m_{\nu_2}$, $m_3^\nu = m_{\nu_3}$, then we have

$$U_\nu = U_{12}^e U_{23}^e U_{\text{MNS}}. \quad (31)$$

Inserting this matrix to Eq. (29), assuming $\sin^2 \theta_{12}^{\text{MNS}} = 1/3$, $\sin^2 \theta_{23}^{\text{MNS}} = 1/2$, we obtain $\sin \theta_{13}^{\text{MNS}} = 0.1487, 0.1130$. For neutrino masses, there appears the following condition

$$\frac{m_{\nu_3}}{m_{\nu_1}} \approx 0.951(0.0518 + \frac{m_{\nu_2}}{m_{\nu_1}}), \quad (32)$$

for $\sin \theta_{13}^{\text{MNS}} = 0.1487$ and

$$\frac{m_{\nu_3}}{m_{\nu_1}} \approx 0.973(0.0281 + \frac{m_{\nu_2}}{m_{\nu_1}}), \quad (33)$$

for $\sin \theta_{13}^{\text{MNS}} = 0.1130$. To be consistent with experiments, only inverted hierarchy is allowed. When Majorana phase is vanishing, the mass spectrum for $\sin \theta_{13} = 0.1487$ is obtained

$$m_{\nu_1} \approx 0.01374[\text{eV}], \quad m_{\nu_2} \approx -0.01378[\text{eV}], \quad m_{\nu_3} \approx -0.01242[\text{eV}]. \quad (34)$$

The sum of neutrino masses is 0.341eV and the effective mass of double beta decay is 35.8meV. Parameters are chosen as $r' \approx -0.9418$, $r'' \approx -0.8767$, $y_1^\nu/y_2^\nu \alpha_7 \approx -0.4364$. For $\sin \theta_{13} = 0.1130$, we have

$$m_{\nu_1} \approx 0.02429[\text{eV}], \quad m_{\nu_2} \approx -0.02433[\text{eV}], \quad m_{\nu_3} \approx -0.02300[\text{eV}]. \quad (35)$$

The sum is 0.460eV and the mass of double beta decay is 49.3meV. Parameters are set as $r' \approx -0.9741$, $r'' \approx -0.9311$, $y_1^\nu/y_2^\nu \alpha_7 \approx -0.4653$.

Similar to the previous section, we input θ_{12}^{MNS} and θ_{23}^{MNS} within 1σ range of [5]. In the above case, we can only have inverted hierarchy for neutrino masses, but in general case, normal hierarchy is also allowed. For inverted mass hierarchy, different parameter space is favoured:

$$\begin{aligned} \Delta m_{31}^2 &= -(2.25 - 2.44) \times 10^{-3} \text{eV}^2, \quad \Delta m_{21}^2 = (7.41 - 7.79) \times 10^{-5} \text{eV}^2, \\ \sin^2 \theta_{12} &= 0.297 - 0.329, \quad \sin^2 \theta_{23} = 0.46 - 0.58. \end{aligned} \quad (36)$$

Giving θ_{12}^{MNS} and θ_{23}^{MNS} , we can get θ_{13}^{MNS} , $\sum m_{\nu_i}$, and $|m_{ee}|$ for each value, shown in Fig. 2. For the case of inverted mass hierarchy, the allowed region is narrow and our prediction is strong. Lower bounds of $\sum m_{\nu_i}$ and $|m_{ee}|$ would be reached by next generation experiments. This model can be also tested by precise measurement of mixing angles. For neutrino masses, $\sum m_{\nu_i}$ and $|m_{ee}|$ are expected with larger values compared to model III. With next-generation experiments of double beta decay and neutrino oscillation, we can have a hint of this model.

4.2 Potential analysis

The superpotential including only the flavon fields is written as

$$\begin{aligned} w_s &= \kappa_1 \chi_1 \chi_2 + \kappa_2 (\chi_3 \chi_6 + \chi_4 \chi_7 + \chi_5 \chi_8) + \lambda_1 \chi_1^3 + \lambda_2 \chi_2^3 + 6\lambda_3 \chi_3 \chi_4 \chi_5 + 6\lambda_4 \chi_6 \chi_7 \chi_8 \\ &\quad + \lambda_5 \chi_1 (\chi_3^2 + \chi_4^2 + \chi_5^2) + \lambda_6 \chi_2 (\chi_6^2 + \chi_7^2 + \chi_8^2) + \eta_1 (\chi_3^2 + \chi_4^2 + \chi_5^2) (\chi_6^2 + \chi_7^2 + \chi_8^2) \\ &\quad + \eta_1' \left(\frac{1}{2} (\chi_4^2 - \chi_5^2) (\chi_7^2 - \chi_8^2) + \frac{1}{6} (-2\chi_3^2 + \chi_4^2 + \chi_5^2) (-2\chi_6^2 + \chi_7^2 + \chi_8^2) \right) \\ &\quad + 4\eta_1'' (\chi_4 \chi_5 \chi_7 \chi_8 + \chi_3 \chi_5 \chi_6 \chi_8 + \chi_3 \chi_4 \chi_6 \chi_7) + \dots, \end{aligned} \quad (37)$$

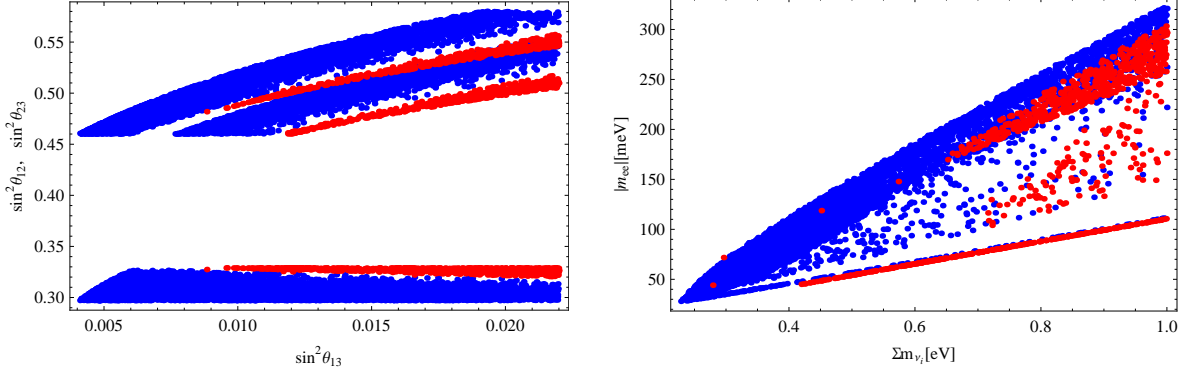


Figure 2: Predicted values of mixing angles (left) and neutrino masses (right). For both figures, blue (dark gray) dots indicate normal mass hierarchy of neutrinos and red (light gray) dots indicate inverted mass hierarchy.

where we omit other fourth couplings which have negative mass dimension. Assuming η_1'' is larger than other negative dimensional operators, the condition of potential minimum becomes

$$\begin{aligned}
\kappa_1 \chi_2 + 3\eta_1 \chi_1^2 + \eta_5 (\chi_3^2 + \chi_4^2 + \chi_5^2) &= 0, \\
\kappa_1 \chi_1 + 3\eta_2 \chi_2^2 + \eta_6 (\chi_6^2 + \chi_7^2 + \chi_8^2) &= 0, \\
\kappa_2 \chi_6 + 6\eta_3 \chi_4 \chi_5 + 2\eta_5 \chi_1 \chi_3 + 4\eta_1'' (\chi_5 \chi_6 \chi_8 + \chi_4 \chi_6 \chi_7) &= 0, \\
\kappa_2 \chi_7 + 6\eta_3 \chi_3 \chi_5 + 2\eta_5 \chi_1 \chi_4 + 4\eta_1'' (\chi_5 \chi_7 \chi_8 + \chi_3 \chi_6 \chi_7) &= 0, \\
\kappa_2 \chi_8 + 6\eta_3 \chi_3 \chi_4 + 2\eta_5 \chi_1 \chi_5 + 4\eta_1'' (\chi_4 \chi_7 \chi_8 + \chi_3 \chi_6 \chi_8) &= 0, \\
\kappa_2 \chi_3 + 6\eta_4 \chi_7 \chi_8 + 2\eta_6 \chi_2 \chi_6 + 4\eta_1'' (\chi_3 \chi_5 \chi_8 + \chi_3 \chi_4 \chi_7) &= 0, \\
\kappa_2 \chi_4 + 6\eta_4 \chi_6 \chi_8 + 2\eta_6 \chi_2 \chi_7 + 4\eta_1'' (\chi_4 \chi_5 \chi_8 + \chi_3 \chi_4 \chi_6) &= 0, \\
\kappa_2 \chi_5 + 6\eta_4 \chi_6 \chi_7 + 2\eta_6 \chi_2 \chi_8 + 4\eta_1'' (\chi_4 \chi_5 \chi_7 + \chi_3 \chi_5 \chi_6) &= 0.
\end{aligned} \tag{38}$$

There are many parameters enough to take independent values for each VEV. To realize the alignment $\chi_3 = \chi_4$ with $\chi_6 \neq \chi_7$, we need a condition $\kappa_2 + 4\eta_1'' \chi_5 \chi_8 = 0$.

5 $\Delta(54)$ model and its stringy origin

Here, we give comments on $\Delta(54)$ models. The $\Delta(54)$ symmetry has a structure similar to S_4 . Indeed, several interesting flavor models have been constructed [107, 108, 109, 110]. Furthermore, the $\Delta(54)$ flavor symmetry as well as D_4 and $\Delta(27)$ can be realized within the framework of heterotic string models on orbifolds [111, 112, 113] and magnetized/intersecting D-brane models [114, 115].² In particular, only triplets as well as a trivial singlet appear as fundamental modes in heterotic orbifold models [112]. From this viewpoint, the model in section 4 is quite interesting.

²See also [116].

We assign $\Delta(54)$ representations and Z_3 charges to the leptons and flavons as shown in Table 5. Those are the same as the model in section 4 except replacing S_4 by $\Delta(54)$. The Z_3 charge assignment is also the same. This Z_3 symmetry plays a role such that different triplet flavon VEVs appear in the mass matrices of the charged leptons and neutrinos. Other symmetries would play the same role in string models.

	$(\ell_e, \ell_\mu, \ell_\tau)$	(e^c, μ^c, τ^c)	$H_{u,d}$	χ_1	χ_2	(χ_3, χ_4, χ_5)	(χ_6, χ_7, χ_8)
$\Delta(54)$	3	3	1	1	1	3	3
Z_3	1	0	0	2	1	2	1

Table 5: Matter content and charge assignment of the $\Delta(54)$ model.

The tensor products of $\Delta(54)$ triplets are the same as those of S_4 . Then, we realize the same superpotential (23) and (24). Thus, we can obtain the same results as one in section 4 when the same vacuum alignment is realized.

The $\Delta(54)$ triplet corresponds to localized fields on three fixed points of the Z_3 orbifold in heterotic models. Thus, the three families of left and right-handed leptons as well as triplet flavons, (χ_3, χ_4, χ_5) and (χ_6, χ_7, χ_8) , would correspond to the modes localized on the three Z_3 fixed points. Since the $\Delta(54)$ trivial singlet corresponds to a bulk mode on the orbifold, the electroweak Higgs fields and singlet flavons, χ_1 and χ_2 , are originated from the bulk modes. Furthermore, VEVs of scalar fields on a fixed point correspond to blow-up of the orbifold singularity. That is, our model suggests that a certain type of blow-up from the orbifold limit to Calabi-Yau manifold would be interesting to derive realistic lepton mass matrices, such that the flavon VEVs corresponding to (25) are realized. Thus, our model would be useful for model building from string models, too.

6 Conclusion

We have studied S_4 models with smaller numbers of flavon fields and free parameters. When we introduce one S_4 triplet flavon, a realistic model cannot be constructed. To be consistent with experiments, we need two triplet S_4 flavons, or one triplet and one doublet at least. By building models with two triplets, or one triplet and one doublet, we have stronger predictions among lepton masses and mixing angles.

Realistic and predictive models are model III and model IV. Both models have seven parameters among six lepton mass eigenvalues and three mixing angles. We have assumed there is a vanishing CP-phase in the lepton sector to make stronger prediction. However, it is easy to extend the models with non-vanishing CP-violation.

We can construct the model with the $\Delta(54)$ flavor symmetry, which is quite similar to model IV. Such a model is quite interesting from the viewpoint of stringy realization. We would study elsewhere on this aspect.

Note to be added

After this paper was completed, Ref. [105] appeared showing the range of the mixing angle θ_{13} in the latest T2K experiment. Our prediction of θ_{13} is compatible with their result.

Acknowledgement

H.I. is supported by Grand-in-Aid for Scientific Research, No.23.696 from the Japan Society of Promotion of Science. T. K. is supported in part by the Grant-in-Aid for Scientific Research No. 20540266 and the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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